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Hence, (3) is true and, therefore, *GDC* is a straight line.

To prove that *G* is the mid-point of *HJ*, we have, from the secants *GD*, *GK*, and *GI*, $GK.GJ=GD.GF=GI.GH$, or $GK.GH=GI.GJ$.

Hence, $GK+GH : GH=GI+GJ : GJ$, or $KH : GH=IJ : GJ$.

$\therefore GH=GJ$, since $HI=JK$ and, therefore, $KH=IJ$.

Also solved analytically by G. B. M. Zerr and J. Scheffer. Professors Zerr and Scheffer obtain the equation of the common chord of the two circles and found that its equation is satisfied by the co-ordinates of *C* and *G*.

CALCULUS.

236. Proposed by J. SCHEFFER, A. M., Hagerstown, Md.

Solve the partial differential equation, $x\dfrac{\partial^2 u}{\partial x^2}+y\dfrac{\partial^2 u}{\partial x\,\partial y}=\dfrac{\partial u}{\partial x}$.

I. Solution by C. EBEN STROMQUIST, Princeton, N. J.

A more general problem, of which the above is a particular case, has been discussed by Darboux (*Leçons sur la Theorie Generale Surfaces*, Vol. 3, p. 53). The solution in this case follows.

Differentiating the given equation with respect to x and setting

$$M=\frac{\partial^2 u}{\partial x^2},\text{ yields }x\frac{\partial M}{\partial x}+y\frac{\partial M}{\partial y}=0\ldots(2).$$

This is a linear differential equation of the first order in M of which a most general solution is* $M=W(x/y)$, where W is an arbitrary function in the argument x/y. Hence

$$w=\int_{x_0}^{x}\int_{x_0}^{x} W\,dx\,dx+x\,w_1(y)+w_2(y)\ldots(3),$$

where w_1 and w_2 are functions of y alone which must be so restricted that w satisfies the given equation.

Since the differential equation from which u is obtained is equation (1) differentiated with respect to x, the result of substituting u in (1) must yield at most a function of y alone. If Y be the function resulting from substituting $u_0=\int_{x_0}^{x}\int_{x_0}^{x} W\,dx\,dx$ in (1), it is readily seen that $Y=0$, and hence that the result of substituting u in (1) gives

$$y\frac{\partial w_1}{\partial y}-w_1=0\ldots(4).$$

Integrating gives $w_1=cy$, whence c is an arbitrary constant.

Substituting in (3), a most general solution of (1) is given by

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*See. for instance, Jordan, *Cours d'Analyse*, Vol. 3, p. 314.

$$u=\int_{x_3}^{x}\int_{x_0}^{x}W(\frac{x}{y})dx\ dx+cxy+w_2(y),$$

where W and w_2 are arbitrary functions in x/y and y, respectively, and c is an arbitrary constant.

II. **Solution by GEORGE W. HARTWELL, Columbia University, New York City.**

Let $\frac{\partial u}{\partial x}=p$. Then the equation will become

$$x\frac{\partial p}{\partial x}+y\frac{\partial p}{\partial y}=p.$$

Solving this by the method of Lagrange we have

$$p=y\,\phi(\frac{x}{y})=\frac{y^2}{y}\,\phi\,(\frac{x}{y}).$$

Let $\phi(\frac{x}{y})$ be the derivative of $\psi(\frac{x}{y})$ with respect to $\frac{x}{y}$. Then

$$\frac{1}{y}\,\phi\,(\frac{x}{y})=\frac{\partial}{\partial x}\,\psi\,(\frac{x}{y}).\quad \therefore\frac{\partial u}{\partial x}=y^2\frac{\partial}{\partial x}\,\psi\,(\frac{x}{y}).$$

Integrating, $u=y^2\,\psi\,(\frac{x}{y})+\chi(y)$.

III. **Solution by G. B. M. ZERR, A. M., Ph. D., Parsons, W. Va.**

Multiply the equation by x and let $x=e^v$, $y=e^w$.

Then $\frac{d^2u}{dv^2}+\frac{d^2u}{dvdw}=2\frac{du}{dw}$. Let $\frac{d}{dv}=D$, $\frac{d}{dw}=D'$. Then $D(D+D'-2)u=0$.

Let $u=e^{av+bw}$. $\therefore\frac{du}{dv}=au$, $\frac{du}{dw}=bu$.

$\therefore a(a+b-2)=0$, therefore, $a=0$, and $a=2-b$.
$\therefore u=\Sigma Ae^{bw}+e^{2v}\Sigma Be^{b(w-v)}=F(w)+e^{2v}f(w-v)=F(y)+x^2f(y/x)$,
where F and f are arbitrary.

Also solved by the Proposer.

MECHANICS.

197. **Proposed by WALTER D. LAMBERT, 416 B Street N. E., Washington, D. C.**

Suppose that a primary planet and its satellite revolve with uniform angular velocity in circular orbits in the same plane. What relation must hold between the radii of their orbits and their angular velocities in order